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The response of dried materials to drying conditions

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Abstract—The response of dried materials to a variation in drying conditions is studied in this paper. The aim of the study is to establish an appropriate system of control of drying processes to avoid destruction of the dried materials. The development of methods of control allows one to design optimal drying processes. We conclude in this paper that the optimal control of drying processes has a wide range of possibilities. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

The deformations or crack formation occurring during drying directly influences the quality of dried products (see refs. [1, 2]). Thus, development of methods of control allowing one to avoid destruction of the dried materials and to design precisely the optimum drying processes, shortening the drying time and saving energy, is very desirable.

The authors found that there is little available literature on this subject and in particular there are very few studies concerning the control of drying processes from the point of view of thermomechanics. The present paper aims to suggest a wide range of possibilities in this area.

The purpose of this paper is to analyze the response of dried materials to a variation in drying conditions. The emphasis is placed upon the drying induced stresses and their control.

We want to show that when the stresses rise, until they reach a critical state which is dangerous for the material, it is possible to reduce them by slowing down the rate of drying. This can be done by altering the parameters of drying, e.g. the temperature, the moisture content or the velocity of flow of the drying medium or all three together.

The considerations in whole are based on the thermomechanical model proposed earlier by Kowalski [3–5], neglecting the phase transition inside the dried body (see ref. [6]).

The problem of the response of dried materials to a variation in drying conditions is demonstrated on a prismatic bar, dried convectively. A two-dimensional problem described by the coupled system of four second-order differential equations is solved with the use of the finite element method for the spatial derivatives and the three-point finite differences for the time derivatives. This numerical method was elaborated by Rybicki [7, 8].

A number of boundary value problems are solved by stable and unstable boundary conditions. In this way we study the reaction of the state of stress in the material to the variation of these conditions.

2. MODEL PRESENTATION

The dried material is assumed to be an elastic porous body with uniformly distributed interconnected pores saturated with liquid (water). Three factors are involved in the deformations:

- alteration of the moisture content Θ;
- alteration of the temperature ϑ;
- stresses induced during drying process σ_{ij}.

Thus, the total strain is a superposition of the moist $\varepsilon_{ij}^{\Theta}$, thermal $\varepsilon_{ij}^{\vartheta}$ and mechanical ε_{ij}^{M} strains:

$$\varepsilon_{ij} = \varepsilon_{ij}^{\Theta} + \varepsilon_{ij}^{\vartheta} + \varepsilon_{ij}^{M} \quad (1)$$

where

$$\varepsilon_{ij}^{\Theta} = \alpha_{\Theta}(\Theta - \Theta_0)\delta_{ij} \quad (2)$$

$$\varepsilon_{ij}^{\vartheta} = \alpha_{\vartheta}(\vartheta - \vartheta_0)\delta_{ij} \quad (3)$$

$$\varepsilon_{ij}^{M} = 2M'\sigma_{ij} + A'\sigma_{kk}\delta_{ij}. \quad (4)$$

The coefficients α_{Θ} and α_{ϑ} express the linear expansion per unit moisture content and per unit temperature, respectively. On the basis of equations (1)–(4) we arrive at the global relation between stresses, strains, moisture content and temperature,

$$\sigma_{ij} = 2M\varepsilon_{ij} + [A\varepsilon - \gamma_{\vartheta}(\vartheta - \vartheta_0) - \gamma_{\Theta}(\Theta - \Theta_0)]\delta_{ij} \quad (5)$$

where M and A are equivalent to Lame constants, $\gamma_{\vartheta} = (2M+3A)\alpha_{\vartheta}$, γ_{Θ}, $\gamma_{\vartheta} = (2M+3A)\alpha_{\Theta}$ and $\varepsilon = \varepsilon_{kk}$.

The stresses have to fulfil the equations of balance of internal forces,

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NOMENCLATURE

A	bulk modulus of dried material [MPa]	α_T	coefficient of convective heat transfer [$W\ m^{-2}\ K^{-1}$]
c_v	specific heat of dried material [$J\ kg^{-1}\ K^{-1}$]	α_ϑ	coefficient of thermal expansion [K^{-1}]
c_ϑ	thermal coefficient of moisture potential [$J\ m^{-3}\ K^{-1}$]	α_Θ	coefficient of moisture expansion
c_Θ	moisture content coefficient of moisture potential [$J\ m^{-3}$]	ε_{ij}	strain tensor
l	latent heat [$J\ kg^{-1}$]	$\boldsymbol{\eta}$	moisture flux vector [$kg\ m^{-2}\ s^{-1}$]
M	shear modulus of dried material [MPa]	ϑ	relative temperature [°C]
$\mathbf{q}$	heat flux vector [$W\ m^{-2}$]	Θ	moisture content of dried material (dry basis)
s	entropy (dry basis) [$J\ kg^{-1}\ K^{-1}$]	Λ_m	coefficient of moisture flow [$kg\ s\ m^{-3}$]
t	time [s]	Λ_T	coefficient of heat conduction [$W\ m^{-1}\ K^{-1}$]
T	absolute temperature [K]	μ	moisture potential [$J\ kg^{-1}$]
u_i	coordinate of the displacement vector [m]	ρ_s	mass density of dried material [$kg\ m^{-3}$]
x, y, z	position coordinates [m]	σ_{ij}	stress tensor [MPa]
Y_a	moisture content of dried medium.	ϕ	relative humidity of dried medium.
Greek symbols			
α_m	coefficient of convective mass transfer [$kg\ s\ m^{-4}$]		

$$\sigma_{ij,j} = 0. \quad (6)$$

The moisture content has to satisfy the equation of moisture mass balance, [4]:

$$\rho_s \dot{\Theta} = -\operatorname{div} \boldsymbol{\eta} \quad (7)$$

where $\boldsymbol{\eta}$ is the moisture flux proportional to the gradient of moisture potential μ,

$$\boldsymbol{\eta} = -\Lambda_m \operatorname{grad} \mu \quad \Lambda_m \geqslant 0. \quad (8)$$

The moisture potential depends on the parameters of state: ε, Θ, ϑ. We take into consideration its linear part,

$$\mu = [c_\vartheta(\vartheta - \vartheta_0) - \gamma_\vartheta \varepsilon + c_\Theta(\Theta - \Theta_0)]/\rho_s \quad (9)$$

where c_ϑ and c_Θ are termed as the thermal and moisture coefficients of the moisture potential.

The temperature has to satisfy the equations resulting from the energy balance equations:

$$\rho_s \dot{s} T = -\operatorname{div} \mathbf{q} \quad (10)$$

where T is the absolute temperature and $\mathbf{q}$ is the heat flux proportional to the gradient of temperature:

$$\mathbf{q} = \Lambda_T \operatorname{grad} \vartheta \quad \Lambda_T \geqslant 0. \quad (11)$$

The entropy s depends on the parameters of state, mentioned above, in the following way [3]:

$$s = [c_v \ln(T/T_r) + \gamma_\vartheta \varepsilon - c_\vartheta(\Theta - \Theta_0)]/\rho_s \quad (12)$$

where c_v is the specific heat of the medium as a whole at constant volume, T_r is a reference temperature and $\vartheta = T - T_r$ is the relative temperature.

The set of relations presented above creates a mathematical model describing the deformations of dried materials. Using the geometrical relation between strains ε_{ij} and displacements u_i:

$$\varepsilon_{ij} = (u_{i,j} + u_{j,i})/2 \quad (13)$$

and by combining the above relations together we get the system of five double coupled, second order differential equations of the model.

3. FORMULATION OF A TWO-DIMENSIONAL PROBLEM

For a two-dimensional problem the displacement of the dried material in one z-direction is assumed to be zero, and all other functions depend on the coordinates x, y and time t, i.e. $u_x = u_x(x, y, t)$, $u_y = u_y(x, y, t)$, $\vartheta = \vartheta(x, y, t)$, $\mu = \mu(x, y, t)$.

The equations of the model reduced to a two-dimensional problem are:

$$\left(2M + A - \frac{\gamma_\Theta^2}{c_\Theta}\right)\frac{\partial^2 u_x}{\partial x^2} + M\frac{\partial^2 u_x}{\partial y^2} + \left(M + A - \frac{\gamma_\Theta^2}{c_\Theta}\right)\frac{\partial^2 u_y}{\partial x\, \partial y} = \left(\gamma_\vartheta - \frac{\gamma_\Theta c_\vartheta}{c_\Theta}\right)\frac{\partial \vartheta}{\partial x} + \frac{\gamma_\Theta}{c_\Theta^\rho}\frac{\partial \mu}{\partial x}$$

$$\left(2M + A - \frac{\gamma_\Theta^2}{c_\Theta}\right)\frac{\partial^2 u_y}{\partial y^2} + M\frac{\partial^2 u_y}{\partial x^2} + \left(M + A - \frac{\gamma_\Theta^2}{c_\Theta}\right)\frac{\partial^2 u_x}{\partial x\, \partial y} = \left(\gamma_\vartheta - \frac{\gamma_\Theta c_\vartheta}{c_\Theta}\right)\frac{\partial \vartheta}{\partial y} + \frac{\gamma_\Theta}{c_\Theta^\rho}\frac{\partial \mu}{\partial y}$$

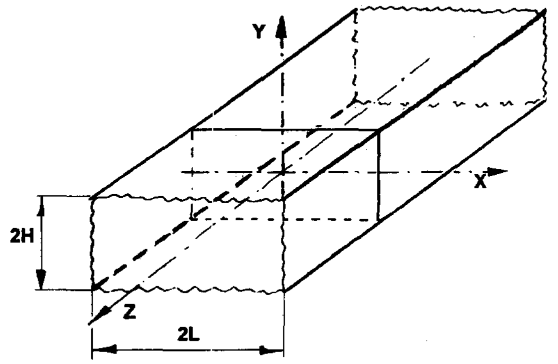

Fig. 1. The dried bar with the rectangular cross-section.

$$K_{\mathrm{m}}\left(\frac{\partial^2\mu}{\partial x^2}+\frac{\partial^2\mu}{\partial y^2}\right)=\frac{\partial\mu}{\partial t}+\gamma_\Theta^\varrho\left(\frac{\partial^2 u_x}{\partial x\,\partial t}+\frac{\partial^2 u_y}{\partial y\,\partial t}\right)-c_\vartheta^\varrho\frac{\partial\vartheta}{\partial t}$$

$$K_{\mathrm{T}}\left(\frac{\partial^2\vartheta}{\partial x^2}+\frac{\partial^2\vartheta}{\partial y^2}\right)=\frac{\partial\vartheta}{\partial t}+K_{\mathrm{E}}\left(\frac{\partial^2 u_x}{\partial x\,\partial t}+\frac{\partial^2 u_y}{\partial y\,\partial t}\right)-K_\Theta\frac{\partial\mu}{\partial t} \tag{14}$$

where

$$K_{\mathrm{m}}=\Lambda_{\mathrm{m}}c_\Theta/\rho_0^2 \quad \gamma_\Theta^\varrho=\gamma_\Theta/\rho_s \quad c_\vartheta^\varrho=c_\vartheta/\rho_s \quad c_\Theta^\varrho=c_\Theta/\rho_s$$

$$K_{\mathrm{T}}=\Lambda_{\mathrm{T}}/c_v^* \quad K_{\mathrm{E}}=T_r(\gamma_\vartheta-c_\vartheta\gamma_\Theta/c_\Theta)c_v^*$$

$$K_\Theta=T_rc_\vartheta/c_\Theta^\varrho c_v^* \quad c_v^*=c_v+T_0c_\vartheta^2/\gamma_\vartheta c_\Theta.$$

We formulate the initial-boundary value problem as follows: find functions u_x, u_y, ϑ and μ which within the rectangle $(-L,L)\times(-H,H)$ and for $t\in R^+$ satisfy the system of equations (14) and the following boundary conditions (Fig. 1): for stresses

$$\sigma_{xx}|_{x=\pm(L+0)}=0 \quad \sigma_{yy}|_{y=\pm(H+0)}=0$$
$$\sigma_{xy}|_{x=\pm(L+0)}=0 \quad \sigma_{xy}|_{y=\pm(H+0)}=0 \tag{15}$$

for the mass exchange

$$\Lambda_{\mathrm{m}}\frac{\partial\mu}{\partial x}\bigg|_{x=\pm L}=\mp\alpha_{\mathrm{m}}(\mu|_{x=\pm L}-\mu_a)$$
$$\Lambda_{\mathrm{m}}\frac{\partial\mu}{\partial y}\bigg|_{y=\pm H}=\mp\alpha_{\mathrm{m}}(\mu|_{y=\pm H}-\mu_a) \tag{16}$$

for the heat exchange

$$\Lambda_{\mathrm{T}}\frac{\partial\vartheta}{\partial x}\bigg|_{x=\pm L}=\pm\alpha_{\mathrm{T}}(\vartheta_a-\vartheta|_{x=\pm L})\mp l\alpha_{\mathrm{m}}(\mu|_{x=\pm L}-\mu_a)$$
$$\Lambda_{\mathrm{T}}\frac{\partial\vartheta}{\partial y}\bigg|_{y=\pm H}=\pm\alpha_{\mathrm{T}}(\vartheta_a-\vartheta|_{y=\pm H})\mp l\alpha_{\mathrm{m}}(\mu|_{y=\pm H}-\mu_a) \tag{17}$$

and under the initial conditions

$$\sigma_{ij}(x,y,0)=0 \quad \mu(x,y,0)=\mu_0 \quad \vartheta(x,y,0)=\vartheta_0. \tag{18}$$

In the above conditions α_{m} and α_{T} denote the coefficients of the convective mass and heat transfer, l is the latent heat of evaporation, μ_a denotes the potential (free enthalpy per unit mass) of the vapour in the surrounding atmosphere (drying medium) and ϑ_a is the temperature of this atmosphere.

Our main aim is to analyse the deformations of dried materials and the drying induced stresses. A number of materials, for example clay, suffer the greatest shrinkage in the first period of drying, sometimes called the 'constant drying rate period' (Fig. 2). We can state this from looking at the shrinkage of the sample's characteristic length L presented in Fig. 2. It can be seen there that the value of shrinkage also depends on the temperature of the drying process.

We consider the first period of drying in which the phase transition of water into vapour takes place at the boundary of the dried material. The temperature of the dried body is kept constant in this period by the stable drying conditions. However, in this paper we study unstable drying conditions, since we want to determine the response of the dried material to the alterations of drying conditions. If we knew this we could control the drying process to avoid the stresses that cause fracture.

The parameters that were altered are the moisture content in air Y and the temperature of the drying medium ϑ_a. Four different drying programs were studied: in the first case both parameters were fixed after some initial period, in the second case the vapour content in air was fixed and the temperature altered; in the third case the temperature was fixed and the vapour content in air altered; in the final case both these parameters were altered.

The system of equations (14) was solved numerically using the finite element method for spatial

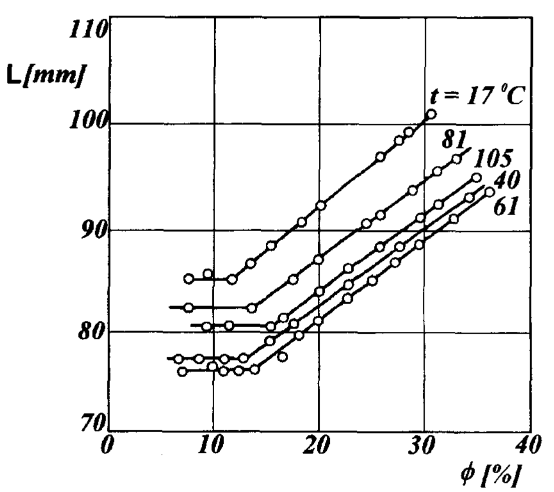

Fig. 2. Curves of shrinkage of a clay at various temperatures [10].

derivatives and the three-point finite differences for the time derivatives. Details can be found in refs. [7, 8, 12]. The material constants were taken from refs. [14, 15].

4. DRYING PROGRAMS AND RESULTS

As can be seen in equation (16), the rate of removal of moisture depends on the difference between the vapour potential at the boundary and that in the drying medium. These potentials can be expressed as follows (see e.g. ref. [16]).

$$\mu_{\partial B} = \mu(p, T_n, x_n) = \mu^{\ominus}(p, T_n) + RT_n \ln x_n,$$

$$\mu_a = \mu(p, T_0, x_a) = \mu^{\ominus}(p, T_n) + RT_a \ln x_a \quad (19)$$

where p is the total pressure of the humid air, T_n and T_a are temperatures of the saturated air at the boundary and of the drying medium, x_n and x_a are the molar ratios of the vapour to the whole gas, in saturated state at the boundary and in the drying medium, respectively. The molar ratios are expressed by the absolute humidity (moisture content in air) as follows (see ref. [17]):

$$x_n = \frac{Y_n}{0.622 + Y_n} \quad \text{and} \quad x_a = \frac{Y_a}{0.622 + Y_a} \quad (20)$$

where Y_n and Y_a are the absolute humidity in the saturated and unsaturated state of the drying medium, respectively.

The difference between the potentials of equation (19) can be written as

$$\mu_{\partial B} - \mu_a = 7.36(\vartheta_a - \vartheta_n) + 8.3\left[\left(T_r - \frac{\vartheta_a + \vartheta_n}{2}\right)\ln\frac{x_n}{x_a} - (\vartheta_a - \vartheta_n)\ln\sqrt{x_a x_n}\right] [\text{J kg}^{-1}] \quad (21)$$

where $\vartheta_n \equiv \vartheta_{\partial B}$ is the temperature of the saturated air at the boundary. Such a developed form of the moisture potential difference was substituted into the boundary conditions, equations (16) and (17).

First drying programme

Each drying programme started from a 30 min preheating period (Fig. 3). The initial parameters of $\vartheta_{0a} = 20°\text{C}$ and $Y_{0a} = 0.015$ kg kg^{-1} are assumed to be in the drying medium (surrounding atmosphere), where Y_{0a} is the maximal moisture content in this medium at the temperature ϑ_{0a}. The temperature of the surrounding atmosphere was then increased up to either $\vartheta_a = 70°\text{C}$ or $\vartheta_a = 50°\text{C}$ and kept constant. Table 1 presents the values of drying parameters.

The moisture content Y was increased from the ideal saturated state Y_{0a} at the temperature ϑ_{0a} up to some maximal value Y_m at the time at which the relative humidity reached value $\phi = 90\%$, and next was decreased linearly to value Y_a at the temperature ϑ_a. The final moisture contents Y_a and the relative humidities ϕ of the drying atmosphere are given in Table 1. The relative humidity of a vapour–air mixture is defined here as:

$$\phi = \frac{Y_a}{0.622 - Y_a}\frac{0.622 + (Y_a)_{max}}{(Y_a)_{max}} = \left(\frac{x_a}{(x_a)_{max}}\right)_{\vartheta_a}. \quad (22)$$

The values of $(Y_a)_{max}$ for a given temperature are taken from the literature (see e.g. ref. [17]).

Figure 4 illustrates the distribution of the moisture potential μ, temperature ϑ, and stresses σ_{yy} along section: $y = 0$, $0 \leqslant x \leqslant L$ in various instants of time. Similar distributions apply to any arbitrary section chosen in the cross-section of the bar (Fig. 1). A difference can be found only in the magnitude of the quantities.

The moisture potential μ, which is constant in the beginning, starts to fall (firstly at the boundary) because of the removal of the moisture. However, with strong heating it can rise to a value which exceeds its initial value (see ref. [11]).

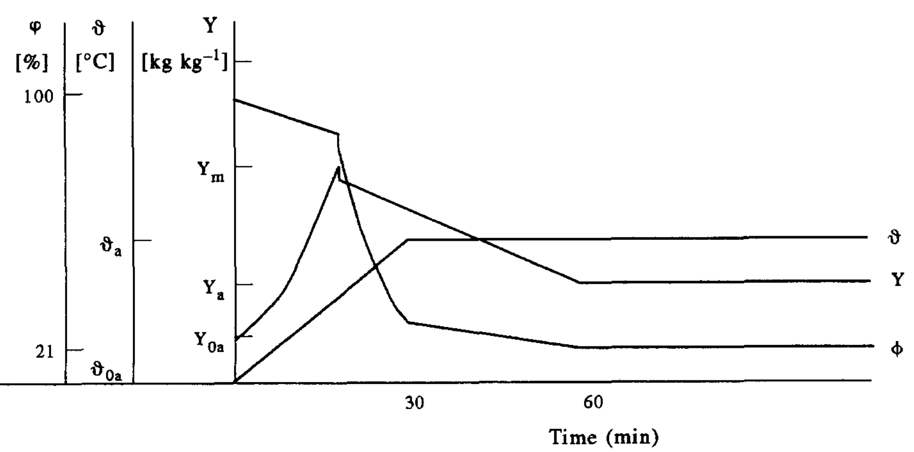

Fig. 3. Drying programme with constant drying parameters after a preheating period.

Table 1.

	ϑ_{0a} [°C]	ϑ_a [°C]	Y_{0a} at ϑ_{0a} [kg kg^{-1}]	$(Y_a)_{max}$ at ϑ_a [kg kg^{-1}]	Y_a [kg kg^{-1}]	ϕ (%)	Maximum stress [MPa]
1	20	70	0.015	0.28	0.043	21	4.03
2	20	70	0.015	0.28	0.052	25	3.52
3	20	50	0.015	0.088	0.0165	21	3.29

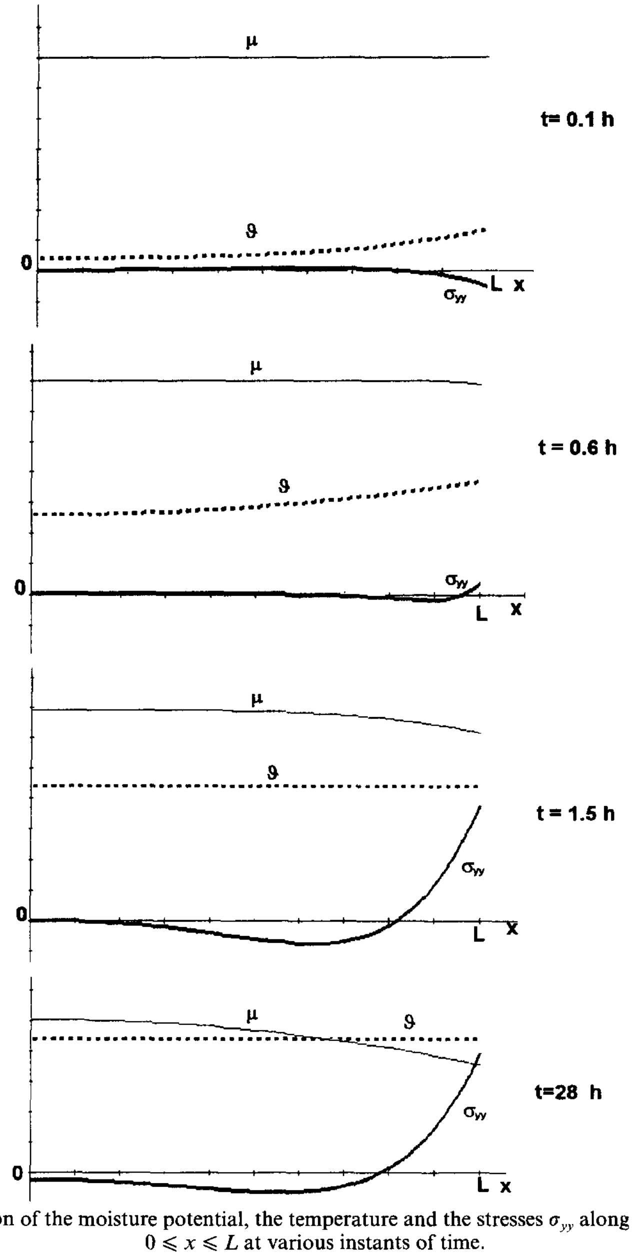

Fig. 4. Distribution of the moisture potential, the temperature and the stresses σ_{yy} along the section $y = 0$, $0 \leqslant x \leqslant L$ at various instants of time.

The temperature of the material ϑ, which was constant in the beginning, starts to increase, after about 90 min it stabilizes reaching the wet bulb temperature.

At first the stresses are negative close to the boundary because there are thermal stresses that dominate during this time. After a short time the stresses are negative inside (compression) and positive close to the boundary (tension), as here the shrinkage due to the removal of the moisture starts to dominate. The maximal tension stress, expected at point $x = L$, $y = 0$,

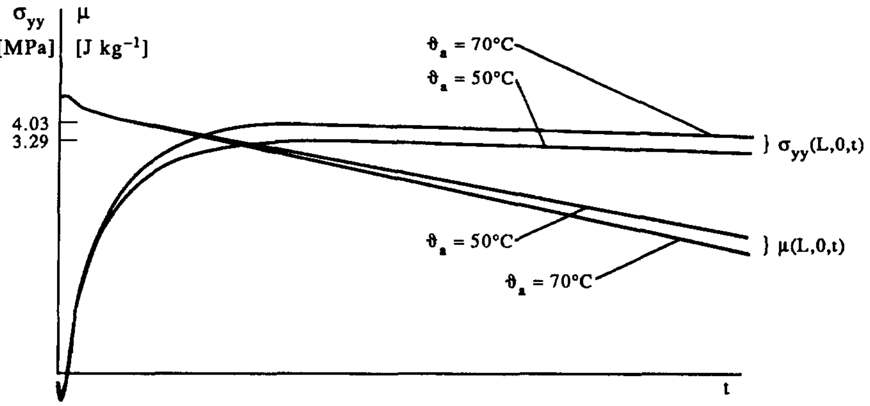

Fig. 5. Stresses σ_{yy} in point $x = L$, $y = 0$.

evidently depends on the temperature and moisture content of the drying medium. As shown in Table 1 the drying programme No. 1, having the highest temperature and the lowest moisture content of the drying medium, involves the biggest stresses.

Figure 5 illustrates the time evolution of stresses at point $x = L$, $y = 0$. We again see that this stress is in the beginning compressive and then after some time tensile. It increases until the time $t_m = 6$ h and then starts to decrease as the moisture distribution in the dried material becomes more homogeneous.

The moisture potential μ in point $x = L$, $y = 0$ of the dried material decreases all the time, but more so for the higher temperature than for the lower.

Second drying programme

The second drying programme deals with the stable moisture content (absolute humidity) $Y_a = 0.052$ and the altered temperature ϑ_a according to the formula:

$$\vartheta_a = \left(1 - 0.4\left|\sin\frac{2\pi t}{3600}\right|\right)\vartheta_A \tag{23}$$

where ϑ_A is 70°C.

It was stated here that the temperature alteration of the drying medium has a significant influence on the parameters of state of the dried material. Figure 7 presents alterations of the stresses σ_{yy} and the moisture potential μ in point $x = L$, $y = 0$ of the dried material and in the time interval $29 \leqslant t \leqslant 32.5$ h caused by the alteration of the temperature ϑ_a of the drying medium. The temperature ϑ_a was altered during this time in the range $42°C \leqslant \vartheta_a \leqslant 70°C$. For such temperatures the stresses σ_{yy} in point $x = L$, $y = 0$ were between $1.92 \leqslant \sigma_{yy} \leqslant 2.28$ [MPa]. The moisture potential in point $x = L$, $y = 0$ was generally diminished in the course of time because of the removal of the moisture, however, since it depends on the temperature (see equation (9)), its periodical increase is observed because of the temperature alteration. Note that the periodical curve of temperature ϑ_a and the periodical curve of stress σ_{yy} have been displaced in phase. The extreme value of σ_{yy} appears later than the extreme value of ϑ_a. The line m illustrates total moisture content in the dried material as a function of time.

Third drying programme

This programme is characterized by constant temperature ϑ_a and altered moisture content Y_A following the formula:

$$Y_a = \left(1 + 0.4\left|\sin\frac{2\pi t}{3600}\right|\right)Y_A \tag{24}$$

where Y_A was chosen to be 0.052 for $\vartheta_a = 70°C$.

Altering the moisture content of the drying medium influences the stresses and the moisture potential of the dried material as does altering the temperature of the medium. Figure 9 illustrates stress $\sigma_{yy}(L, 0, t)$, moisture potential $\mu(L, 0, t)$, moisture content in the drying medium $Y_a(t)$, and moisture content in the dried material $m(t)$ in the time interval $29 \leqslant t \leqslant 32.5$ h.

It can be seen that the phase displacement between functions $\sigma_{yy}(L, 0, t)$ and $Y(t)$ is about 1/2 h. Thus the stress is minimal when the moisture content reaches maximum, this means an instantaneous reduction of the stress when the moisture content in the drying medium is increased.

For the moisture content alterations in the range $0.052 \leqslant Y_a \leqslant 0.073$ the stresses were altered between $3.10 \geqslant \sigma_{yy} \geqslant 2.95$ [MPa]. A detailed inspection of the stress curve indicates a decrease of the average value of stress $\sigma_a = (\max \sigma_{yy} + \min \sigma_{yy})/2$ in time. This is because of the homogenization of the moisture distribution in this stage of drying.

The moisture potential μ also has a periodical character with average values decreasing with time.

Similarly the temperature of the dried body is also of periodical character, in spite of the fact that the temperature of the drying medium is kept constant. This phenomenon is linked with the latent heat and the wet bulb temperature. The latter depends on the moisture content of the drying medium. The temperature alteration of dried material in this programme of drying is small, for example, point $x = L$,

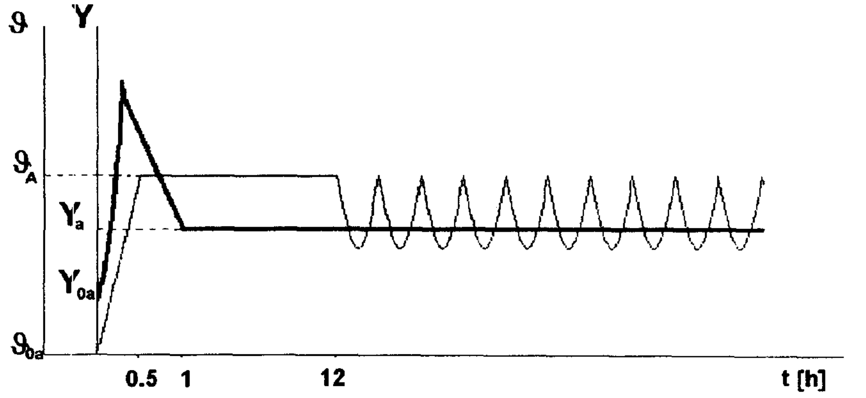

Fig. 6. Drying programme with altered temperatures of the drying medium.

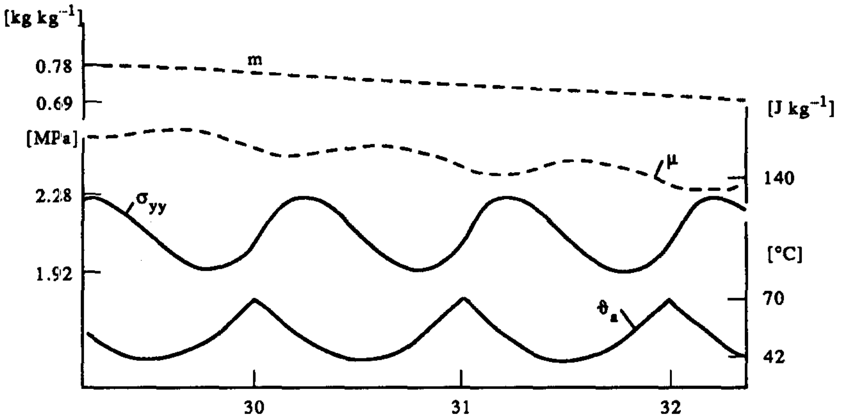

Fig. 7. Stress σ_{yy} and moisture potential μ in point $x = L$, $y = 0$ of the dried material and the temperature ϑ_a of the drying medium vs time.

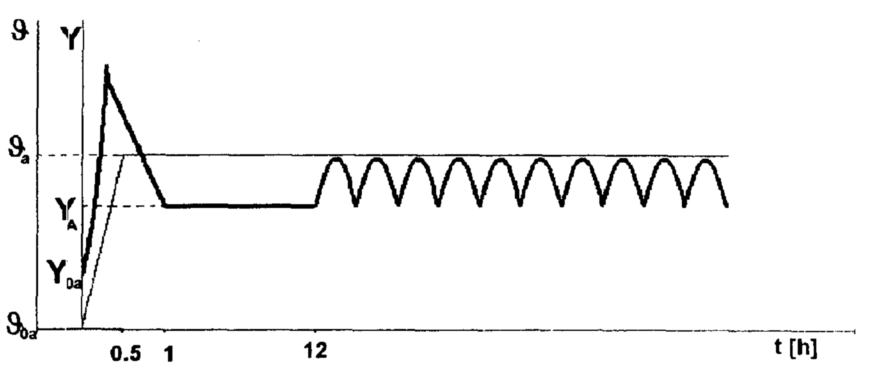

Fig. 8. Drying programme with altered moisture content of the drying medium.

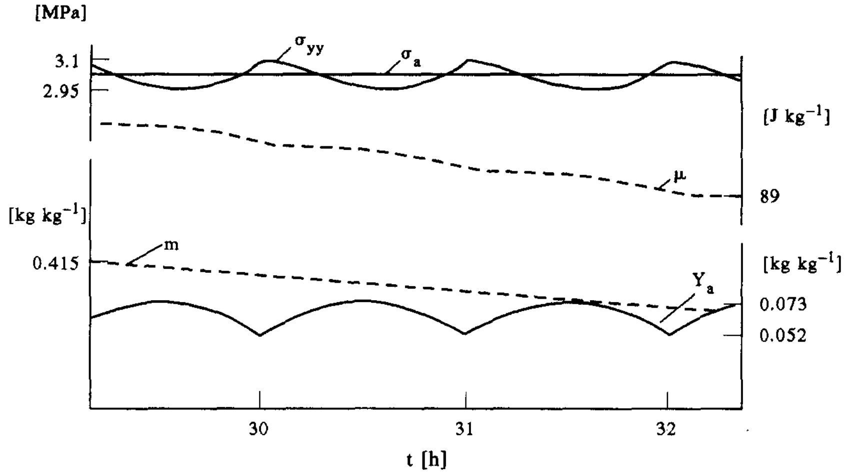

Fig. 9. Stress σ_{yy} and the moisture potential μ in point $x = L$, $y = 0$ of the dried material and the moisture content Y_a of the drying medium vs time.

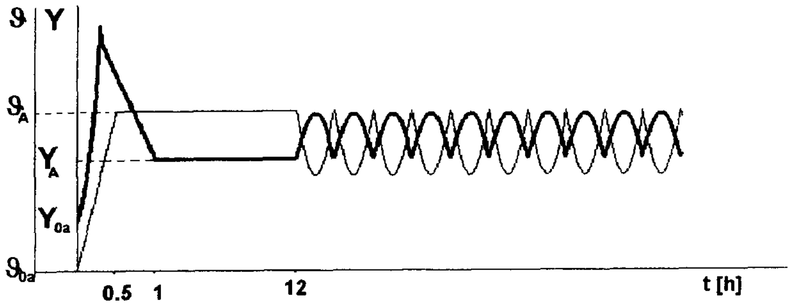

Fig. 10. Drying programme where the temperature and the moisture content of the drying medium were altered in opposite phase.

$y = 0$ is in range: $65.53°C \leqslant \vartheta(L, 0, t) \leqslant 65.66°C$, therefore, it is not shown in Fig. 9.

The line m illustrates the total moisture content vs time. It can be seen that the material in the third drying programme is dried more quickly than in the second one.

The above considerations lead to the conclusion that both the temperature alterations and moisture content alterations of the drying medium allow for stress reduction. The question arises of how to use these possibilities to design a drying process in order to obtain a short drying time and to avoid the destruction of dried material.

Fourth drying programme

Finally, we come to the results of our calculations for the situation where both the temperature and the moisture content of the drying medium were altered simultaneously. We have considered here two cases: firstly, when the temperature and the moisture content are altered in accordance with their phases and secondly, when they are altered in opposite phases. Figure 11 illustrates the stress at point $x = L$, $y = 0$ for the first case. It is obvious that an increase in the temperature ϑ_a accelerates the drying process and on the other hand an increase of the moisture content Y_a slows it down. Therefore, the response of the dried material to these alterations is not as big as in the case when the temperature ϑ_a and the moisture content Y_a are altered in opposite phases, see Fig. 12. In the second case the effects summarize the above.

The line m in Figs. 11 and 12 illustrate the rates of drying—that is the total amount of moisture removed from the dried material per unit time. It is visible that the drying rate is faster in the case when $\vartheta_a(t)$ and $Y(t)$ are in opposite phases.

5. CONCLUSIONS

(1) The mechanical state of dried materials is significantly dependent on the drying parameters, such as the temperature and the moisture content of the drying medium. This gives rise to the possibility of controlling the mechanical state of dried material and

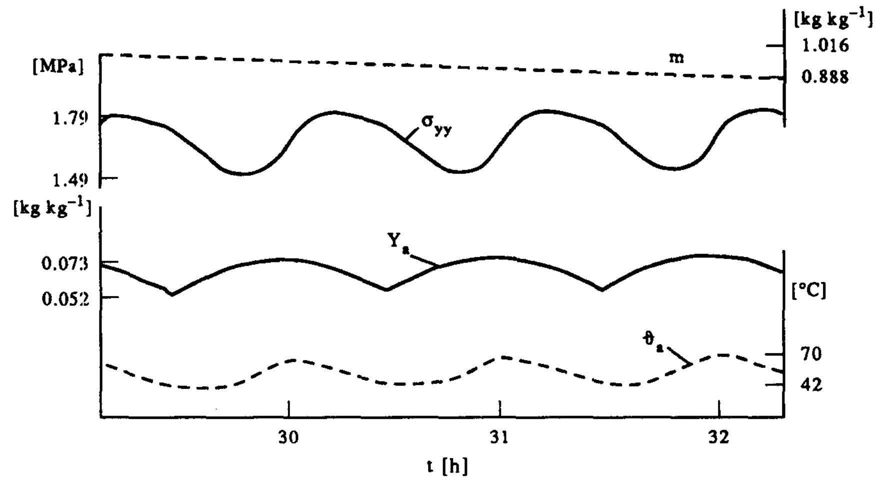

Fig. 11. Stress σ_{yy} at point $x = L$, $y = 0$ vs time for the temperature and the moisture content of the drying medium altered with the same phases.

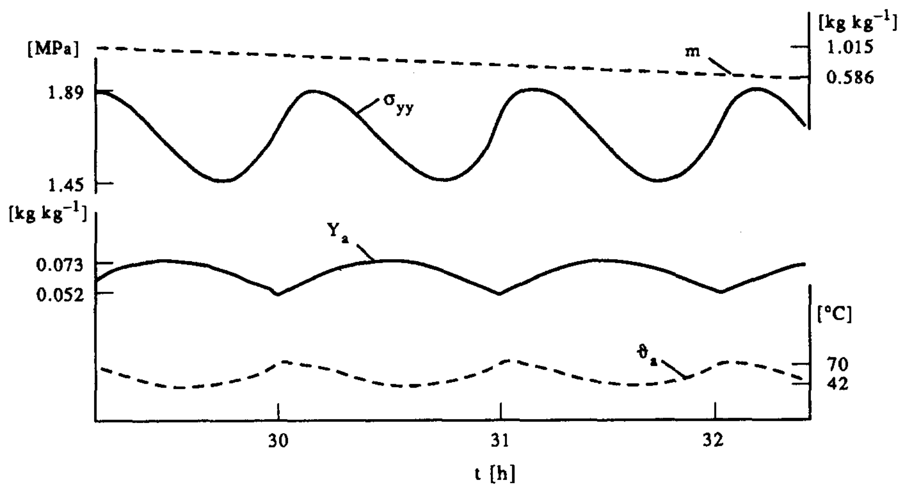

Fig. 12. Stress σ_{yy} at point $x = L$, $y = 0$ vs time for the temperature and the moisture content of the drying medium altered with opposite phases.

in particular the drying induced stresses, by appropriate alteration of these parameters.

(2) The response of the dried material to the alteration of temperature ϑ_a is subject to some retardation whereas the response to the alteration of moisture content Y_a is almost instantaneous.

(3) Comparison of the second and the third drying programme leads to the statement that the undesirable shrinkage caused by the removal of the moisture from the boundary of the material is alleviated by thermal expansion. This effect worsens in the second drying programme where the temperature $= \vartheta_a$ and hence the temperature of the dried material decreases. This explains why the stress amplitude in the second drying programme was 0.18 [MPa] while in the third drying programme it was only 0.075 [MPa]. The rate of drying seems to be better in the third drying programme.

(4) Inspection of the drying programmes allows one to state that the average stress $\sigma_a = (\max \sigma_{yy} + \min \sigma_{yy})/2$ at point $x = L$, $y = 0$ decreases more quickly when the temperature of the drying medium is of periodical character.

The general conclusion is that the control of drying parameters gives a wide range of possibilities for designing optimum drying processes more precisely in order to avoid the destruction of dried materials and at the same time to optimize drying time and energy consumption.

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